BOARD QUESTION PAPER : JULY 2017
MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions: **(6) [12]**

i. The inverse of the matrix $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ is ______.

(A) $\frac{1}{5}\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ (B) $\frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

(C) $\frac{1}{5}\begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$ (C) $\frac{1}{5}\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$

ii. If $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\bar{c} = -5\hat{i} + 2\hat{j} + 3\hat{k}$, then $\bar{a} \cdot (\bar{b} \times \bar{c})$ = _______.

(A) 100 (B) 101
(C) 110 (D) 109

iii. If a line makes angles 90°, 135°, 45° with the X, Y, and Z axes respectively, then its direction cosines are _______.

(A) $0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ (B) $0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

(C) $1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (D) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(B) Attempt any THREE of the following: **(6)**

i. If the line $\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\bar{r} \cdot (3\hat{i} - 2\hat{j} + p\hat{k}) = 10$, find the value of p.

ii. If a line makes angles α, β, γ with co-ordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$.

iii. Write the negations of the following statements:

a. $\forall\ n \in N, n + 7 > 6$

b. The kitchen is neat and tidy.

iv. Find the angle between the lines whose direction ratios are 4, –3, 5 and 3, 4, 5.

v. If $\bar{a}, \bar{b}, \bar{c}$ are position vectors of the points A, B, C respectively such that $3\bar{a} + 5\bar{b} - 8\bar{c} = \bar{0}$, find the ratio in which A divides BC.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, then find the value of 'x'.

ii. Write the converse, inverse and contrapositive of the following statement. "If it rains then the match will be cancelled."

iii. Find p and q, if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of prependicular lines.

(B) Attempt any TWO of the following: **(8)**

i. Find the equation of the plane passing through the intersection of the planes $3x + 2y - z + 1 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).

ii. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio m : n, then prove that $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m + n}$. Hence find the position vector of R which divides the line segment joining the points A(1, –2, 1) and B(1, 4, –2) internally in the ratio 2 : 1.

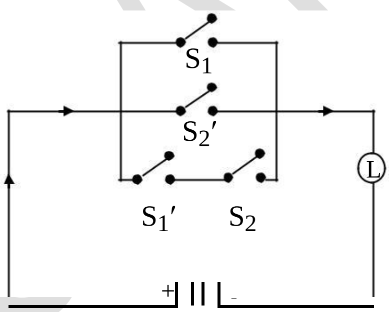
iii. The angles of the ΔABC are in A.P. and b : c = √√ 3: √√ 2 then find ∠A, ∠B, ∠C.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the vector equation and cartesian equation of a line passing through the points A(3, 4, –7) and B(6, –1, 1).

ii. Find the general solution of $\cot x + \tan x = 2 \operatorname{cosec} x$.

iii. Express the following switching circuit in symbolic form of logic. Construct its switching table and write your conclusion from it:



(B) Attempt any TWO of the following: **(8)**

i. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ verify that A (adj A) = | A | I.

ii. A company manufactures bicycles and tricycles each of which must be processed through machines A and B. Machine A has maximum of 120 hours available and machine B has maximum of 180 hours available. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B.
If profits are ` 180 for a bicycle and ` 220 for a tricycle, formulate and solve the L.P.P. to determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.

iii. If θ is the measure of acute angle between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$, then prove that

$$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|, a + b \neq 0$$

Hence find the acute angle between the lines

$x^2 - 4xy + y^2 = 0$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. Given $f(x) = 2x, x < 0$

$= 0, x \geq 0$

then $f(x)$ is _______.

(A) discontinuous and not differentiable at $x = 0$

(B) continuous and differentiable at $x = 0$

(C) discontinuous and differentiable at $x = 0$

(D) continuous and not differentiable at $x = 0$

ii. If $\int_0^{\alpha}(3x^2 + 2x + 1)\,dx = 14$, then $\alpha =$ _______.

(A) 1 (B) 2

(C) –1 (D) –2

iii. The function $f(x) = x^3 - 3x^2 + 3x - 100, x \in R$ is _______.

(A) increasing (B) decreasing

(C) increasing and decreasing (D) neither increasing nor decreasing

(B) Attempt any THREE of the following: **(6)**

i. Differentiate 3^x w.r.t. $\log_3 x$

ii. Check whether the conditions of Rolle's theorem are satisfied by the function $f(x) = (x - 1)(x - 2)(x - 3), x \in [1, 3]$

iii. Evaluate: $\int \frac{\sqrt{\tan x}}{\sin x . \cos x}\,dx$

iv. Find the area of the region bounded by the curve $x^2 = 16y$, lines $y = 2$, $y = 6$ and Y-axis lying in the first quadrant.

v. Given X – B (n, p)

If n = 10 and p = 0.4, find E(X) and var (X).

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. If the function $f(x) = \frac{(5^{\sin x} - 1)^2}{x \log(1 + 2x)}$, for $x \neq 0$ is continuous at $x = 0$, find f (0).

ii. The probability mass function for X = number of major defects in a randomly selected appliance of a certain type is

X = x	0	1	2	3	4
P(X = x)	0.08	0.15	0.45	0.27	0.05

Find the expected value and variance of X.

iii. Suppose that 80% of all families own a television set. If 5 families are interviewed at random, find the probability that
   a. three families own a television set.
   b. at least two families own a television set.

(B) Attempt any TWO of the following: **(8)**

i. Find the approximate value of cos (60° 30′)
   (Given: $1° = 0.0175^c$, sin 60° = 0.8660)

ii. The rate of growth of bacteria is proportional to the number present. If, initially, there were 1000 bacteria and the number doubles in one hour, find the number of bacteria after 2½ hours.
   [Take $\sqrt{2} = 1.414$]

iii. Prove that:
$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx, \quad \text{if } f(x) \text{ is an even function.}$$
$$= 0, \quad \text{if } f(x) \text{ is an odd function.}$$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. If f (x) is continuous on [–4, 2] defined as
$$f(x) = 6b - 3ax, \quad \text{for } -4 \le x < -2$$
$$= 4x + 1, \quad \text{for } -2 \le x \le 2$$
   Show that $a + b = -\frac{7}{6}$

ii. If u and v are two functions of x, then prove that
$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dv}\int v\,dx\right]dx$$

iii. Probability distribution of X is given by

X = x	1	2	3	4
P(X = x)	0.1	0.3	0.4	0.2

   Find $P(X \ge 2)$ and obtain cumulative distribution function of X.

(B) Attempt any TWO of the following: **(8)**

i. Solve the differential equation $\frac{dy}{dx} - y = e^x$
   Hence find the particular solution for $x = 0$ and $y = 1$.

ii. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and
$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}, \text{ where } \frac{dy}{dx} \ne 0$$
   Hence if $y = \sin^{-1}x$, $-1 \le x \le 1$, $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
   then show that $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$, where $|x| < 1$.

iii. Evaluate: $\int \frac{8}{(x+2)(x^2+4)}\,dx$